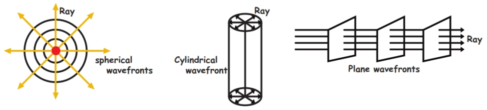
WAVE OPTICS

Wave Front



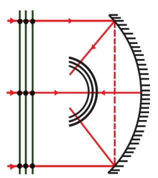
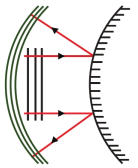
Point light source → spherical wavefront
Linear light Source → cylindrical wavefront
Source at infinity → Plane wave front

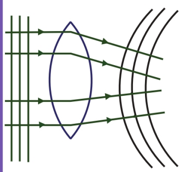
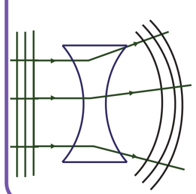
Huygen's principle

i) Each point on a wavefront acts as a fresh source of new disturbance, called secondary waves or wavelets.
The secondary wavelets spread out in all directions with the speed of light in the given medium.

ii) A common envelope or common tangent to these secondary wavelets at any later time gives secondary wavefront at that time

Reflection and refraction of wavefronts

	Incident wavefront	Reflected wavefront
Concave Mirror 	Plane wavefront	Spherical converging wavefront
Convex Mirror 	Plane wavefront	Spherical diverging wavefront

	Incident wavefront	Refracted wavefront
Convex Lens 	Plane wavefront	Spherical converging wavefront
Concave Lens 	Plane wavefront	Spherical diverging wavefront

Phase Difference & Path Difference

$\Phi = \frac{2\pi}{\lambda} \Delta x$

Phase Difference & Time Difference

$\Phi = \frac{2\pi}{T} \Delta t$

Resultant Amplitude

$Y_1 = A_1 \sin \omega t$ and
$Y_2 = A_2 \sin(\omega t + \Phi)$

Resultant $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Phi}$

- $\cos\Phi = 1 \Rightarrow A = A_{max} = \sqrt{(A_1 + A_2)^2} = A_1 + A_2$
- $\cos\Phi = -1 \Rightarrow A = A_{min} = \sqrt{(A_1 - A_2)^2} = A_1 - A_2$

$\frac{A_{max}}{A_{min}} = \frac{A_1 + A_2}{A_1 - A_2}$

- Intensity $\propto$ (amplitude)^2

Resultant Intensity

We have, $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Phi$

- $\cos\Phi = 1 \Rightarrow I = I_{max}$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

- $\cos\Phi = -1 \Rightarrow I = I_{min}$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$I_{max} \propto A^2_{max}$ & $I_{min} \propto A^2_{min}$

$\frac{I_{max}}{I_{min}} = \frac{A^2_{max}}{A^2_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$

If $I_1 = I_2 = I_0$
$\Rightarrow I = 4I_0 \cos^2\frac{\Phi}{2}$

YDSE in Liquid

When YDSE setup is immersed in a liquid, there is change in wavelength

$n = \frac{c}{v} = \frac{v\lambda}{v\lambda'} = \frac{\lambda}{\lambda'}$ n → refractive index

$\lambda_{medium} = \lambda' = \frac{\lambda}{n}$ or $\lambda' = \frac{\lambda}{\mu}$ μ → refractive index

In air $Y_n = n\frac{D\lambda}{d}$

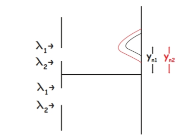
In medium $Y'_n = \frac{nD\lambda'}{d} = \frac{nD\lambda}{d\mu}$

Fringe width in air $\beta = \frac{D\lambda}{d}$

In medium $\beta' = \frac{D\lambda}{\mu d} \Rightarrow \beta_{med} = \frac{\beta_{air}}{\mu}$

$\Rightarrow \beta_{med} < \beta_{air}$

Overlapping



Let n_1^{th} max of λ_1 wavelength overlaps with n_2^{th} max of λ_2 wavelength

$Y_{n1} = Y_{n2}$

$\frac{n_1 D\lambda_1}{d} = \frac{n_2 D\lambda_2}{d}$

$n_1\lambda_1 = n_2\lambda_2$

- As we move further away, then overlapping of colours increases if white light is used
- At larger distance, all colours again overlap to give white light pattern

Introduction Of Thin Transparent Sheet in YDSE

Optical path length and geometrical path length

Refractive index $\mu = \frac{c}{v}$

$\mu = \frac{v\lambda}{v\lambda_m} \Rightarrow \lambda_m = \frac{\lambda}{\mu}$

Time taken by light to travel x length in medium,

$t = \frac{x}{c/\mu} = \frac{\mu x}{c}$

Distance travelled by light in vaccum in same time = optical path length

Optical Path Length (OPL)=velocity x time $= c \times \frac{\mu x}{c} = \mu x$

If Geometrical Path Length (GPL) = x, then OPL=μx, where μ is the refractive index of the medium

Young's Double-slit experiment (YDSE)

Path difference $\Delta X = \frac{y_n d}{D}$

In general $\Delta X = \frac{y_n d}{D}$

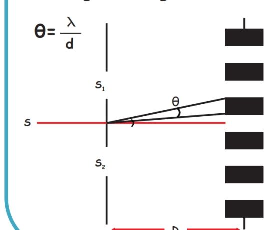
Distance of Minima and Maxima from Central maximum

Maxima: $Y_n = \frac{nD\lambda}{d}$ n = 0,1,2...

Minima: $Y_n = (2n-1)\frac{D\lambda}{2d}$ n = 1,2...

Angular fringe width

$\theta = \frac{\lambda}{d}$



Constructive and destructive interference

Constructive interference	Destructive interference
Phase difference at the point of observation $\Phi = 0°$ or $2n\pi$, n = 0,1,2,.....	$\Phi = 180°$ or $\Phi = (2n-1)\pi$; n=1,2...
Also, $\Delta x = n\lambda$, n = 0,1,2,.....	Also, $\Delta x = (2n-1)\frac{\lambda}{2}$, n = 1,2,3,.....
Resultant intensity at the point of observation is maximum	Resultant intensity at the point of observation will be minimum
$I = I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$	$I = I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

Intensity at any point on screen

For all maxima $I = 4I_0$ (If $I_1 = I_2 = I_0$)
For all minima, I = 0
Note:
Fringe visibility $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Fringe Width or Band width (β)

$\beta_{dark} = \frac{D\lambda}{d}$

$\beta_{bright} = \frac{D\lambda}{d}$

For interference pattern $\beta_{dark} = \beta_{bright} = \frac{D\lambda}{d}$

WAVE OPTICS

02

Introduction of thin transparent sheet

Path difference $\Delta x = s_2P - s_1P$

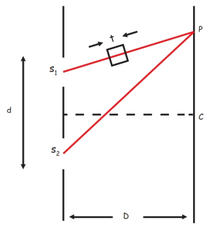
Additional path difference = $(\mu - 1)t$

Geometrical path difference before inserting sheet, $\Delta x = \frac{yd}{D}$

$y = \frac{D}{d}\Delta x$

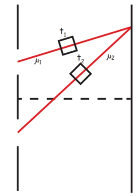
After introducing sheet, $y' = \frac{D}{d}[\Delta x + (\mu - 1)t]$

Shift $S = y' - y = \frac{D}{d}(\mu - 1)t$



If two plates are introduced,

Shift $S = \left|(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2\right| \frac{D}{d}$



Interference of reflected light :
For normal incidence $r = 0$ so, $2\mu t = (2n-1)\frac{\lambda}{2}$

Interference of refracted light :
For normal incidence $2\mu t = n\lambda$

Single Slit Diffraction

Path difference = $\Delta x = d \sin\theta$

- Formation of first secondary minima
  Path difference $= \frac{\lambda}{2}$
- Formation of 2ndsecondary minima
  Path difference = 2λ
- Formation of n^{th} secondary minima
  $d \sin\theta_n = n\lambda$
  n=1,2,3,......

First secondary maxima

But the intensity of 1^{st} secondary maxima is lower than central maximum

N^{th} secondary Maxima

$x = (2n+1)\frac{\lambda}{2}$

n = 1,2,3....

$d \sin\theta_n = (2n+1)\frac{\lambda}{2}$

Ratio of intensities of central maxima and secondary maximas

$1 : \frac{1}{2} : \frac{1}{61} : \frac{1}{121} : ....$

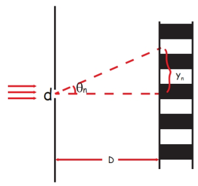
Distance of N^{th} secondary maxima from CM

$\theta_n = (2n+1)\frac{\lambda}{2d}$

n = 1,2,3.....

$y = (2n+1)\frac{D\lambda}{d}$

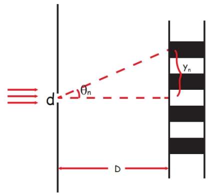
n = 1,2,3.....



Distance of N^{th} secondary minima from CM

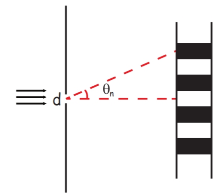
$y = \frac{nD\lambda}{d}$

n = 1,2,3.....



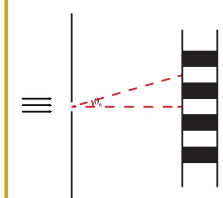
Angular position of N^{th} secondary minima

$\theta_n \approx \frac{n\lambda}{d}$

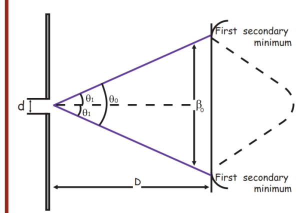


Angular position of N^{th} secondary Maxima

$\theta_n \approx (2n+1)\frac{\lambda}{2d}$



Angular width and linear width of central maximum



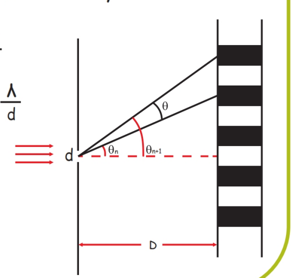
$\beta_0 = \theta_0 D = \frac{2D\lambda}{d}$

Angular width and linear width of secondary maxima

Angular position of n^{th} minimum, $\theta_n = n\frac{\lambda}{d}$

Angular position of $n+1^{th}$ minimum, $\theta_{n+1} = n+1\frac{\lambda}{d}$

Linear width, $\beta = \frac{D\lambda}{d}$



Angular width and Linear width of secondary minima

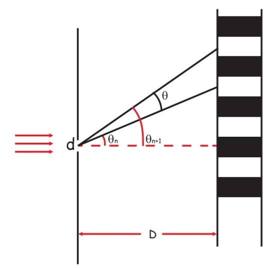
Angular position of n^{th} maximum $\theta_n = (2n+1)\frac{\lambda}{2d}$

Angular position of $(n+1)^{th}$ maximum $\theta_{n+1} = (2(n+1)+1)\frac{\lambda}{2d}$

$\theta = \frac{\lambda}{d}$

Linear width $\beta = \theta D$

$\beta = \frac{D\lambda}{d}$



Validity of Ray Optics: Fresnel's Distance

$Z_F = \frac{d^2}{\lambda}$

Resolving Power (R.P) of a microscope

$\frac{1}{d} = \frac{2n \sin\theta}{\lambda}$

Resolving Power

$R.P = \frac{1}{\text{limit of resolution}}$

Resolving power of a telescope

$R.P = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$

Law of Malus

$I \propto \cos^2\theta$

$I = I_0 \cos^2\theta$

When $\theta = 0°$ or $180°$, $\cos\theta = \pm 1 \Rightarrow I = I_0$

When $\theta = 90°$, $\cos\theta = 0 \Rightarrow I = 0$

Polarisation by Reflection

Brewster found that at the polarising angle ,the reflected and transmitted rays are perpendicular to each other

$n = \tan i_p$

This is Brewster's Law